Multi-Target Detection and Tracking in a Heterogeneous Environment with Multiple Resource-Constrained Sensors

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We consider the multiple-sensor multiple-target detection and tracking problem in a sensor network consisting of resource-constrained multi-function radars. Tracking and detection takes place in a known heterogeneous environment that causes signal degradation. The proposed formulation includes the effects of the environment in a multi-function radar model for searching, detection, tracking, and resource usage. The sensor-target assignment problem is optimized to maximize information gain subject to the radar resource constraints. Simulations demonstrate the effects of including the environment in the radar modeling and show performance improvement compared to performing assignment without considering the environment.

Nomenclature

A	=	assignment matrix
a_{ij}	=	assignment of sensor i to target j
E	=	expectation function
Λ	=	environmental impedance scalar field
λ	=	line of sight environmental effects
γ	=	sector area environmental effects
η	=	sensor resource constraint
F	=	process model
Γ	=	mutual information gain
h	=	measurement model
H	=	linearized measurement model
i	=	information state contribution
I	=	information matrix contribution
Y	=	information matrix
y	=	information state
p	=	probability
ρ	=	range
P	=	state covariance matrix
Q	=	process noise covariance
R	=	measurement noise covariance
T_{track}	=	time on target
T_{search}	=	time searching
$\hat{x}$	=	target state estimate
z	=	measurement vector

I. Introduction

Multiple-target tracking (MTT) is necessary for a wide range of applications such as surveillance, air traffic management, combat scenarios, and other types of monitoring. The problem of MTT consists of obtaining measurements

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from multiple targets and then accurately estimating the states of each target simultaneously. In instances where the targets are not assumed indistinguishable, data association and track-to-track fusion algorithms have been posed such as a probabilistic data association filter [1] and multiple hypothesis tracking [2]. In [3], a clustering algorithm is used to improve track-to-track fusion performance. The problem of target detection is also often considered along with track-to-track fusion. When including detection, the number of targets tracked by the sensor network changes over time and the sensors must detect a target with some probability before it is tracked. Research focusing on the detection aspect of the problem can be found in [4–6]. In [7] a scenario of low-detection probability is considered and the target assignment strategy is such that the probability of detection of each tracked target is maximized.

Another area of study is reduction in computational complexity. Because the general assignment problem is NP complete, as the number of targets and sensors in a system increase, the computational requirements can quickly become intractable. Various algorithms and heuristics to improve efficiency of the generalized assignment problem have been proposed [8–11]. One that is applied to the MTT problem [12] uses an algorithm based on linear matrix inequalities to reduce the computational load.

In sensor networks, there are many advantages to having a distributed control and decision-making framework. Centralized planners often require unrealistic computational requirements and are less robust than decentralized networks. Optimizing the resource consumption of a radar network is possible through choice of sensor-target assignment. In [13], a distributed network topology was considered and each sensor solved the optimal assignment for itself, and then a binary consensus algorithm determined a global assignment. However, this optimization was only over the resource usage. In [14], the problem was formulated as a cooperative game and each sensor determined its own assignment based off of a distributed computation of the best response dynamics. However, this work only optimized over tracking accuracy. In [15], the tracking performance and resource usage are jointly optimized, but the resource usage is optimized to minimize the low probability of intercept and detection is not considered.

Due to their nearly instantaneous control, multi-function radars are well suited to the problem of multi-target tracking. In fact, the multi-function radar is a commonly used and well studied sensor for the MTT problem [16–18]. In scenarios that cover large areas or require the sensing of a large number of targets, networks of multi-function radars can be used to meet sensing requirements. A multi-function radar has the ability to search for new targets and to track existing targets, while having a finite amount of resources to utilize in total. One of the primary factors affecting the performance of a multi-function radar sensor network is the allocation of resource usage. At any given instant in time, each radar is assigned a set of targets to track, and its remaining resources are devoted to searching the surrounding area for additional targets. The assignment of radars to targets determines the overall resource allocation for a sensor network. One particular scenario that is relevant and stresses a sensor network's resources is the raid scenario, where there are many more targets than sensors and each sensor must track multiple targets. In this scenario, in order to properly detect new targets that enter the region of interest, radars must have sufficient available resources dedicated to searching. Therefore optimizing the sensor to target assignment, and thus the overall resource usage, is paramount to performance in raid-like scenarios. Furthermore, in combat scenarios, for a weapon to prosecute a target, the tracking accuracy of that target must meet some minimum requirement. For this requirement to be met on a given target, sufficient resources must be available to gain a sufficiently small uncertainty estimate of that target.

While there has been a lot of research into optimizing the MTT problem by networks of multi-function radars, one extension to the multi-target tracking and detection problem that has not been considered is including the effects of the environment on sensor performance. Realistically, sensors are affected by the environment, e.g., via radar ducting, environmental impedance, weather effects, etc. However, these effects are not typically considered in system modeling. There may be cases where knowledge of the conditions of the surrounding environment is available to the sensors. If the effects of the environment are known, the detection probabilities and target-tracking algorithms could use the knowledge of the environment to produce better assignments and estimates.

This paper considers both target detection and tracking. Specifically, a target must be detected by having a signal that is above some threshold of probability of detection before it may be tracked, and the sensors may also lose track of the target if the probability of false alarm of a signal grows too high. However, it is assumed that the targets are distinguishable from each other and data association or track to track fusion are not necessary. The sensor network is also assumed to be distributed. Theoretical radar performance models are utilized to characterize the resource usage of a radar for searching and tracking. Tools from signal detection theory are used to model the probability distributions of signal amplitudes and to classify detections as hits or false alarms. For tracking, a decentralized extended information filter is implemented to estimate the states of targets from range and bearing measurements and to provide a means of quantifying the value of sensor-target pairings.

The contributions of this paper are as follows: (1) a mathematical model of multi-target detection and tracking by a resource-constrained multi-function radar network operating in a heterogeneous environment; and (2) a target-assignment framework for maximizing information subject to radar resource constraints. The optimization problem is solved numerically and the simulations demonstrate the benefit of considering the environment in target assignment. By modeling environmental effects and including their consideration in the assignment optimization, lower variance tracking estimates are produced and the sensor resources are more efficiently used. In scenarios such as raid scenarios that stress the sensor network's resources, this performance improvement would be particularly valuable.

The outline of this paper is as follows. Section II reviews the fundamentals of the information filter and the assignment problem. Section III introduces radar performance models for search and track and shows how a signal-disrupting environment is incorporated. Section IV poses the constrained optimized assignment problem by formulating an objective function and resource constraints. Section V offers a conclusion and suggests future directions for this research.

II. Background

This section reviews the formulation and solution process for the information filter and the general assignment problem which will be used to estimate and optimize the multi-target tracking problem.

A. Information Filter

A convenient filtering technique to estimate multi-sensor multi-target tracking problems is the information filter, also known as the inverse covariance form of the Kalman filter. The derivation can be found in [19]. By operating in the information space, sensor data can be fused by summing the information contribution of each sensor. This filter operates in information space rather than state space. Let m and n denote discrete time steps. The information form is obtained by transforming the state estimate $\hat{\mathbf{x}}$ and associated covariance matrix $\mathbf{P}$ to the information state $\hat{\mathbf{y}}$ and information matrix $\mathbf{Y}$ as follows [19]:

$$\mathbf{Y}(m|n) = \mathbf{P}^{-1}(m|n) \tag{1}$$
$$\hat{\mathbf{y}}(m|n) = \mathbf{P}^{-1}(m|n)\hat{\mathbf{x}}(m|n) = \mathbf{Y}(m|n)\hat{\mathbf{x}}(m|n). \tag{2}$$

With the defined information quantities, the traditional Kalman filter can then be expressed in information space. The information propagation coefficient is denoted by $\mathbf{L}$. The prediction step is [19]

$$\mathbf{L}(k|k-1) = \mathbf{Y}(k|k-1)\mathbf{F}(k)\mathbf{Y}^{-1}(k|k-1) \tag{3}$$
$$\mathbf{Y}(k|k-1) = [\mathbf{F}(k)\mathbf{Y}^{-1}(k-1|k-1)\mathbf{F}(k)^T + \mathbf{Q}(k)]^{-1} \tag{4}$$
$$\hat{\mathbf{y}}(k|k-1) = \mathbf{L}(k|k-1)\hat{\mathbf{y}}(k-1|k-1), \tag{5}$$

where $\mathbf{Q}$ is the process noise covariance.

The information state contribution of a measurement and associated information matrix are defined as [19]

$$\mathbf{i}(k) = \mathbf{H}(k)^T\mathbf{R}^{-1}\mathbf{z}(k) \tag{6}$$
$$\mathbf{I}(k) = \mathbf{H}(k)^T\mathbf{R}^{-1}\mathbf{H}(k). \tag{7}$$

The update step sums the information state and matrix with the information state contribution and associated matrix [19].

$$\mathbf{Y}(k|k) = \mathbf{Y}(k|k-1) + \mathbf{I}(k) \tag{8}$$
$$\hat{\mathbf{y}}(k|k) = \hat{\mathbf{y}}(k|k-1) + \mathbf{i}(k) \tag{9}$$

The information filter can be extended to nonlinear observation models by using the innovation term ν and the Jacobian of the measurement model ∇_x to compute the information contribution as

$$\nu(k) = \mathbf{z}(k) - \mathbf{h}(\hat{\mathbf{x}}(k|k-1)) \tag{10}$$
$$\mathbf{I}(k) = \nabla h_x^T(k)\mathbf{R}^{-1}\nabla h_x(k) \tag{11}$$
$$\mathbf{i}(k) = \nabla h_x^T(k)\mathbf{R}^{-1}[\nu(k) + \nabla h_x(k)\hat{\mathbf{x}}(k|k-1)]. \tag{12}$$

For multiple sensors, the information filter can be applied to the case where the sensor network is decentralized. For a decentralized multi-sensor multi-target tracking problem, if each node has the same initial estimate, the network is fully connected, and communication occurs at each time step, then each node will have the same estimate of the global state after each time step. These are assumptions made here. As a result, the prediction step can occur at each node concurrently, and the predictions made by each node will be equivalent. Thus (3)-(5) remain the same for the decentralized case. After each node makes its measurement, it computes the information contribution and communicates it to the rest of the network. The update step sums over the information contribution state and matrix obtained from each node's measurement. Let index i refer to a sensor and index j refer to a target. For a sensor i updating its estimate of target j with all measurements of j, the update computation is expressed as

$$\mathbf{Y}_{i,j}(k|k) = \mathbf{Y}_{i,j}(k|k-1) + \sum_{i=1}^{N} \mathbf{I}_{i,j}(k) \tag{13}$$

$$\hat{\mathbf{y}}_{i,j}(k|k) = \hat{\mathbf{y}}_{i,j}(k|k-1) + \sum_{i=1}^{N} \mathbf{i}_{i,j}(k). \tag{14}$$

Because the only information each sensor must communicate to other sensors is its information contributions, which consists of just a vector and matrix of numbers, the communications requirement for a fully connected network remains reasonably low.

B. Assignment Problem

For a given set of n sensors and m targets, there are 2^{nm} possible sensor-target pairings when there are no constraints on the number of targets a sensor may be assigned to or the number of sensors that may be assigned to a target. Each different pairing will provide some different level of utility. In this sense, there exists some some optimal assignment of sensors to targets. For instance, one sensor might have a better estimate of a particular target than another. The assignment problem consists of determining the optimal assignment of sensors to targets.

Consider a set of sensing agents $S = \{s_1, ..., s_n\}$ and a set of target agents $T = \{t_1, ..., t_m\}$. The utility of sensor i being assigned to target j is defined as a function $J_{i,j} = J(s_i, t_j)$. The assignment of sensor i to target j can be described by a matrix A where each entry in the matrix $A(i, j) = a_{i,j} \in [0, 1]$ is

$$a_{ij} = \begin{cases} 0 \text{ if sensor } i \text{ not assigned to target } j \\ 1 \text{ if sensor } i \text{ assigned to target } j \end{cases} \tag{15}$$

The assignment problem objective function can then be expressed as follows:

$$J = \sum_{i=1}^{N} \sum_{j=1}^{M} J_{i,j} a_{i,j}. \tag{16}$$

The selection of $a_{i,j}$ that maximize the value function J is an optimal solution to the assignment problem. The assignment selection may also be constrained to certain selections. The constrained assignment problem will be used to form and solve the optimal assignment of radars to targets subject to constraints representing resource usage.

III. Operating a multi-function radar in a heterogeneous environment

This section considers the performance and resource usage model of a multi-function radar in searching and tracking and incorporate the effects of a heterogeneous signal degrading environment into that model.

A. Search function

The performance of a multi-function radar can be described by its signal-to-noise ratio (SNR). When performing a search, the SNR of a radar [20], defined as Ψ_s, can be described by

$$\Psi_s = \frac{P_{avg} A_e \xi T_{search}}{4\pi\kappa T_0 L_s \rho^4 \Omega}, \tag{17}$$

where P_{avg} is the average transmitted power, A_e is the antenna area, ξ is the radar cross section, κ is the Boltzman constant, T_0 is the radar temperature, L_s represents system losses associated with search, and T_{search} is the search time within range ρ, and solid angle Ω.

A convenient way to represent radar resource usage is time. To quantify radar resource usage for a search, (17) can be rearranged and solved for T_{search}. The parameters of the radar for searching can be considered as constant and expressed as a single parameter α_s. Assume that Ω is constant and included in α_s. The search function of radar i can be described as

$$T^i_{\text{search}} = \alpha_s \Psi L_s \rho_i^4. \tag{18}$$

This formulation assumes that some desired SNR and search range is selected and yields the required resource usage of a radar to achieve that SNR over the desired search area. Through the system losses term, we incorporate a signal-disrupting heterogeneous environment into the radar search resource usage model.

We model the environment as a scalar field, where each point in the field represents some magnitude of radar signal disruption or impedance at that point in space, which could be the result of atmospheric conditions, rain, etc. We define the scalar field function, which takes a position coordinate (x, y) and returns the scalar at that point in the field, as

$$\Lambda = \Lambda(x, y).$$

To include the environmental impact in the resource model, we assume that the system losses L_s is an affine function of the environment in the sector that the radar is searching. Assume that the radar searches the sector uniformly across Ω. The total effect of the environment for searching in a search sector can then be approximated by taking the area integral of Λ over that sector, i.e.,

$$\gamma_i = \iint \Lambda(x, y) d\rho d\Omega. \tag{19}$$

L_s can then be computed as $L^i_s = L_s(\gamma_i) = \beta\gamma_i + \bar{L_s}$, where $\bar{L_s}$ represents the losses assuming no environmental affects and β_s is a parameter that characterizes the change in system losses with respect to changes in environmental impedance. The radar resource usage for searching a given sector can therefore be expressed as a function of the sector size, the desired SNR, and the environmental conditions within the search sector.

To model detection when a target enters the search sector of a radar, the radar will begin to measure a signal amplitude corresponding to the target in addition to a measured range and bearing. We append the signal amplitude to the measurement vector to implement the detection function. As is commonly used in radar modeling[1], assume the received signal amplitude has a Rayleigh probability density distribution. Taking q as the detected signal amplitude, in the case of noise only, the probability density as a function of the detected amplitude is

$$p_n(q) = \frac{q}{\sigma^2} \exp\left[\frac{-q^2}{2\sigma^2}\right], \tag{20}$$

where σ is the variance of the background noise. In the presence of a target, we can write the probability density function as

$$p_t(q) = \frac{q}{\sigma^2(1+d)} \exp\left[\frac{-q^2}{2\sigma^2(1+d)}\right], \tag{21}$$

where $1 + d = \Psi_s$ is the expected SNR of the target The expected SNR in the probability modeling is the same SNR chosen in the radar resource usage modeling in (18).

To consider environmental effects, the variance in the detected signal amplitude is a function of the environmental effects, i.e., $\sigma(\gamma_i) = \sigma_\gamma \gamma_i + \bar{\sigma}$, where $\bar{\sigma}$ is the variance with no environmental effects and σ_γ is a constant parameter characterizing the rate of change of the variance as a function of the environmental noise. From these two probability density functions, a signal amplitude can be drawn with or without a target in a heterogeneous signal disrupting environment. A thresholding value τ can be chosen such that any detected amplitude greater than τ is classified as a hit, and any detected amplitude less than τ is classified as a miss. τ may be chosen to achieve a sufficiently low false alarm rate based off of the expected SNR and σ.

B. Track function

A similar process can be followed to model the resource usage of a radar tracking a target. The performance for radar tracking is the SNR Ψ_t for tracking [20] expressed as

$$\Psi_t = \frac{P_t G^2 \chi \xi T_{\text{track}}}{(4\pi)^3 \kappa T_0 L_s \rho^4}, \tag{22}$$

where κ, ξ, T_0, ρ are as before, P_t is the peak transmitter power, G is the antenna gain, χ is the wavelength, L_t is the system losses associated with tracking, and T_{track} is the time the target is in track. As before, we compute radar usage for sensor i tracking target j by solving for the tracking time, which yields

$$T_{\text{track}}^{(ij)} = \alpha_t \Psi_t L_t \rho_{ij}^4 \tag{23}$$

where α_t consists of the parameters of the radar for tracking. As before, we assume that the desired SNR can be selected and in part determines the resources required for tracking.

To model the effects of the environment on radar tracking performance, assume that the total effect of the environment can be approximated by the line integral from the sensor to the target over the scalar field representing environmental effects. The total environmental impedance from sensor i to target j is then

$$\lambda_{ij} = \int_{S_{ij}} \Lambda(x, y) dS, \tag{24}$$

where S_{ij} is the line from sensor i to target j. Assume that L_t is a function of λ, $L_t^{(ij)} = L_t(\lambda_{ij}) = \beta_t \lambda_{ij} + \bar{L_t}$, where $\bar{L_t}$ is the losses with no environment and β_t is the rate of change in losses with respect to the change in environmental impedance. The total resource usage for tracking a target can then be expressed as a function of the desired SNR, the environment between the sensor and target, and the distance between sensor and target.

While a target is being tracked, the target's state is estimated with a decentralized extended information filter. We define the state space of a single target as

$$\mathbf{x} = \begin{bmatrix} x & \dot{x} & y & \dot{y} \end{bmatrix}^T.$$

The target dynamics are represented by the following second order discrete time model

$$\mathbf{x}_j(k+1) = F\mathbf{x}_j(k) + \mathbf{w}_j, \tag{25}$$

where

$$F = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{26}$$

$\mathbf{w}_j$ is assumed to be zero-mean Gaussian noise modeling the unknown acceleration of the target and has the form

$$E\{\mathbf{w}_j(k)\mathbf{w}_j^T(k)\} = Q = \begin{bmatrix} \frac{1}{4}\Delta t^4 & \frac{1}{2}\Delta t^3 & 0 & 0 \\ \frac{1}{2}\Delta t^3 & \Delta t^2 & 0 & 0 \\ 0 & 0 & \frac{1}{4}\Delta t^4 & \frac{1}{2}\Delta t^3 \\ 0 & 0 & \frac{1}{2}\Delta t^3 & \Delta t^2 \end{bmatrix} \sigma_a^2, \tag{27}$$

where σ_a is the assumed variance of the acceleration of the target.

The measurement model of sensor i measuring target j is as follows:

$$\mathbf{z}_{i,j}(k) = \mathbf{h}_k(\mathbf{x}_j, \mathbf{v}_{ij}) = \begin{bmatrix} \sqrt{(x_j(k) - x_i(k))^2 + (y_j(k) - y_i(k))^2} \\ \arctan \frac{x_j - x_i}{y_j - y_i} \end{bmatrix} + \mathbf{v}_{ij} = \begin{bmatrix} \rho_{ij} \\ \theta_{ij} \end{bmatrix}, \tag{28}$$

where $\mathbf{v}_{ij}(k)$ is the sensor noise of sensor i measuring target j. $\mathbf{v}_{ij}(k)$ is zero mean uncorrelated Gaussian noise, $E\{\mathbf{v}_{ij}(k)\} = \mathbf{0}_{2x1}$, and has variance

$$E\{\mathbf{v}_{ij}(k)\mathbf{v}_{ij}^T(k)\} = \begin{bmatrix} \sigma_{\rho,ij}^2 & 0 \\ 0 & \sigma_{\theta,ij}^2 \end{bmatrix} = \mathbf{R}_i.$$

The linearized observation model for sensor i sensing target j is as follows:

$$H(k) = \nabla h_x(k) = \begin{bmatrix} \frac{x_j(k)-x_i(k)}{\sqrt{(x_j(k)-x_i(k))^2+(y_j(k)-y_i(k))^2}} & \frac{y_j-y_i}{\sqrt{(x_j(k)-x_i(k))^2+(y_j(k)-y_i(k))^2}} \\ \frac{y_j-y_i}{(x_j(k)-x_i(k))^2+(y_j(k)-y_i(k))^2} & \frac{x_i-x_j}{(x_j(k)-x_i(k))^2+(y_j(k)-y_i(k))^2} \end{bmatrix}. \tag{29}$$

For further insight and simplicity, (29) can also be expressed in the observation coordinates as

$$H_{ij}(k) = \begin{bmatrix} \sin\theta_{ij} & \cos\theta_{ij} \\ \frac{\cos\theta_{ij}}{\rho_{ij}} & -\frac{\sin\theta_{ij}}{\rho_{ij}} \end{bmatrix}. \tag{30}$$

The expected information gain at each time step can then be written as follows, with the indices omitted for brevity:

$$I(k) = H^T(k)R^{-1}H(k) = \begin{bmatrix} \frac{\sin^2\theta(k)}{\sigma_\rho} + \frac{\cos^2\theta(k)}{\rho^2\sigma_\theta} & \frac{\sin\theta(k)\cos\theta(k)}{\sigma_\rho} - \frac{\sin\theta(k)\cos\theta(k)}{\rho^2\sigma_\theta} \\ \frac{\sin\theta(k)\cos\theta(k)}{\sigma_\rho} - \frac{\sin\theta(k)\cos\theta(k)}{\rho^2\sigma_\theta} & \frac{\sin^2\theta(k)}{\rho^2\sigma_\theta} + \frac{\cos^2\theta(k)}{\sigma_\rho} \end{bmatrix}. \tag{31}$$

The expected information gained from a measurement is a function of the expected variance of the measurement as well as the distance between sensor and target. Assume that $\sigma_{\rho,ij}$ changes according to the losses resulting from the environment characterized by λ_{ij}, thus $\sigma_{\rho ij} = \sigma_\rho(\lambda_{ij})$. We again take this to be a function in affine form, i.e., $\sigma_{\rho,ij} = \sigma_\lambda\lambda_{ij} + \bar{\sigma}_\rho$. For simplicity, assume the variance σ_θ is constant.

For m targets, the information state and matrix of the system can be extended to estimate the state of all tracked targets as follows:

$$\hat{\mathbf{y}}(k|k) = \begin{bmatrix} \hat{\mathbf{y}}_1(k|k) \\ \vdots \\ \hat{\mathbf{y}}_m(k|k) \end{bmatrix}, \ \mathbf{Y}(k|k) = \begin{bmatrix} \mathbf{Y}_1(k|k) & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{Y}_m(k|k) \end{bmatrix}, \tag{32}$$

where $\hat{\mathbf{y}}_j(k|k)$ and $\mathbf{Y}_j(k|k)$ are the information state and matrix of the jth target, respectively. Under the assumptions made in section Section II.A, each node is able to independently perform the estimation of the global state by summing all the information contributions from other nodes at each time step.

To illustrate the effects of the inclusion of the environment in the model, consider a simple scenario in which a single sensor detects and tracks a single target according to the described model. The simulation is performed for the case where the environment is homogeneous and the case where the environment is heterogeneous. For this scenario and all other scenarios utilizing a heterogeneous environment presented in this paper, the following function was used:

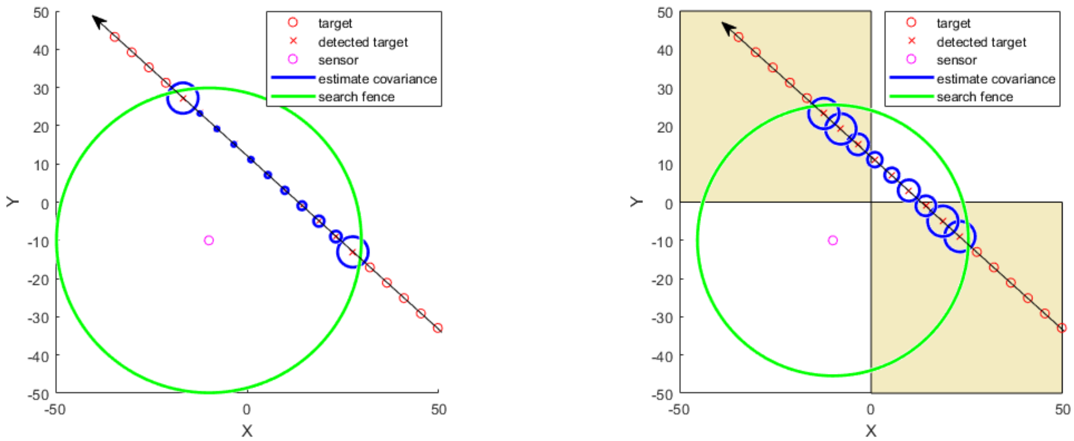
$$\Lambda(x, y) = \begin{cases} 0.1, xy \geq 0 \\ 1.0, xy < 0 \end{cases}. \tag{33}$$

Figure 1 depicts the track of the target as it moves around a sensor. The locations where the target is tracked and undetected is differentiated. The size of the search fence of the sensor, which is taken as constant in this example, is also shown.

This example illustrates a few key effects of including the environment. First, the search fence to maintain proper SNR for detection becomes smaller, because it requires more time to properly search the portions of the search sector with environmental impedance. Second, the target is detected one time step later and loses track of the target one step earlier in the model with the heterogeneous environment. Third, the estimated covariance of the position estimate is larger when the target is in parts of the environment with higher impedance. If the environment is known, each of these effects can be anticipated and leveraged to yield better sensor-target pairings.

IV. Optimized Resource Constrained Sensor-Target Assignment

This section defines resource constraints on the radar and the objective function to optimize the sensor-target assignment. Simulations are performed to demonstrate the effects of the modeling in Section III on the optimization.



(a) Homogeneous environment **(b) Heterogeneous environment**

Fig. 1 Target detection and tracking compared across a homogeneous and heterogeneous environment. Due to environmental effects, (b) shows a smaller search fence, larger covariance estimates, and less time in track compared with (a)

A. Resource Constraint

In a given duty cycle, a radar can be assigned to track for some portion of it and search for another portion. However, these allocations are limited by some maximum resource limit available to the radar [21]. The total resource radar i uses at any given time is

$$T_{\text{total}}^{(i)} = T_{\text{search,i}} + \sum_{j=1}^{M} T_{\text{track}}^{(i,j)} a_{i,j}.$$

If each radar has maximum available resource $T_{\max}$, then the resource usage at any time step $T_{\text{total}}^{(i)}$, which is a function of the search area and the targets it is tracking, must be less than or equal to the maximum available. Assume that at each time step all radar resources are used, that is $T_{\text{total}}^{(i)} = T_{\max}$. Also assume that resources not being used for tracking targets are devoted to searching, i.e., $T_{\text{search}}^{(i)} = T_{\max} - \sum_{j=1}^{M} T_{\text{track}}^{(i,j)} a_{i,j}$.

To define a constraint that appropriately balances T_{search} and T_{track}, assume that there is a minimum allowable sized search fence for each radar, characterized by $\rho_{\min}$. Depending on the environment, there is an associated resource requirement $T_{\text{search,min}}$ to fully search the sector defined by $\rho_{\min}$. To ensure that the radar has sufficient resources to search the minimum area, we require that the difference between the maximum available resources and the resources devoted to tracking is less than the minimum search resource requirement. This constraint can also be expressed as follows: $T_{\max} - \sum_{j=1}^{M} T_{\text{track}}^{(i,j)} a_{i,j} \leq T_{\text{search,min}}$. For convenience of notation, we rewrite this constraint as

$$\eta_i = \frac{T_{\max} - \sum_{j=1}^{M} T_{\text{track}}^{(i,j)} a_{i,j}}{T_{\text{search,min}}} \leq 1. \tag{34}$$

We further constrain the assignment so that a target may only be assigned a single sensor. (A sensor, however, is allowed to track multiple targets, so long as it satisfies the resource constraint.) While there may be some utility of multiple sensors being assigned to a single target to produce a better estimate of that target, it would have to be weighed against the utility lost from reducing the search area to redundantly track a target. This comparison is not considered. This constraint may be expressed as a linear matrix inequality. With the assignment matrix $A(i, j) = a_{ij}$, the number of targets M, and the number of sensors N, the constraint can be expressed as

$$A^T \mathbf{1}^{N\times 1} \leq \mathbf{1}^{M\times 1}, \tag{35}$$

where **1** is a vector with one in every entry.

A second simple scenario was simulated to demonstrate the effects of resource usage and is shown in Fig. 2. Two targets moved through the search space of a single sensor and were both detected and tracked. The detection and tracking of the targets is illustrated. Additionally, the search fence at the time of detection of each target is shown. This scenario illustrates the reduction in search area that results from tracking additional targets. When the first target is detected and tracked, the search area decreases significantly. The search area increases as the first target gets closer to the sensor at a rate proportional to the quartic of the range. When the second target is detected, the search fence is reduced again.

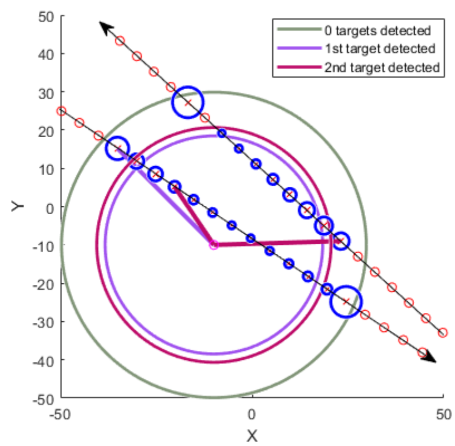


Fig. 2 Resource usage during multi-target tracking. As the radar tracks more targets, the size of the sector it can search decreases.

B. Target Tracking Objective Function

The use of the information filter provides a convenient metric for the value of a measurement, i.e., the information matrix. At a given time step, a sensor is able to evaluate (31) with the predicted state estimate, providing an estimate of the information gained by taking a measurement. The higher the information gained for a measurement, the lower the resulting variance of the estimate after the measurement will be. The sensor-target assignment that provides the estimates with the lowest variance also maximizes the information gain. The information gain is thus a valid objective function to optimize the sensor-target assignment. The expected information matrix from observation can be computed by (31), and the magnitude of the expected mutual information gain can then be written as

$$\Gamma_{ij}(k) = \frac{1}{2} \log \frac{|\mathbf{Y}_{ij}(k|k-1) + I_{ij}(k)|}{|\mathbf{Y}_{ij}(k|k-1)|}. \tag{36}$$

We use this quantity to quantify the value gained by pairing sensor i with target j. Thus the global value for any given assignment scheme can be computed as

$$V = \sum_{i=1}^{N} \sum_{j=1}^{M} \Gamma_{i,j} a_{i,j}. \tag{37}$$

Taking the form of (16), the optimization of the sensor-target assignment then consists of finding the value of A that maximizes V.

This objective function allows for the system to remain decentralized. As noted in Section II.A, due to the assumptions made, $\mathbf{Y}_{ij}(k|k-1)$ can be computed for each sensor target pairing by each sensor individually. As a result, Γ_{ij} can also be computed for each sensor target pairing by each sensor individually. Thus each sensor can perform the optimization individually and arrive at the same assignment solution as every other sensor in the network. This enables a globally optimal solution to be found by each sensor simultaneously in a decentralized manner.

The resource-constrained assignment optimization problem can then be expressed as

$$A^* = \arg\max_A \left[\sum_{i=1}^{N} \sum_{j=1}^{M} \Gamma_{i,j} a_{i,j} \right], \tag{38}$$

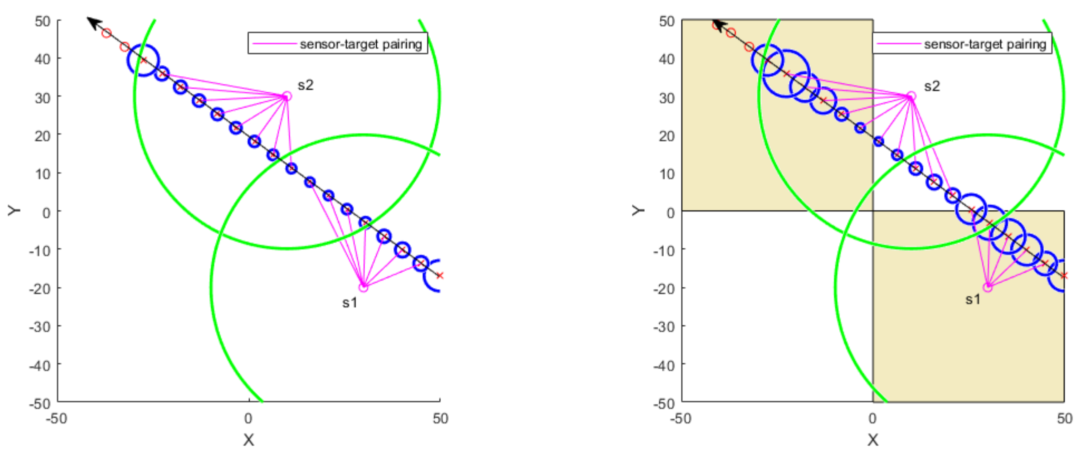
subject to the resource constraint of maximum radar usage

$$\eta_i \leq 1, \quad i = 1, ..., N. \tag{39}$$

C. Simulation Results

The search and track operations of the radar network is performed as follows. 1) For all tracked targets, the prediction step of the information filter is performed. 2) The assignment optimization subject to resource constraints is performed by brute force. Any targets not assigned to a sensor is removed from the state. 3) Measurements are taken according to the optimal assignment, the information is shared across the network, and the update step of the information filter is performed. 4) The search fence for each radar is computed from the remaining resources available to that radar. 5) Each radar measures target signals that are within its search fence. 6) Each target signal is classified as a detection or a false alarm. 7) The newly detected targets are added to the filter state.

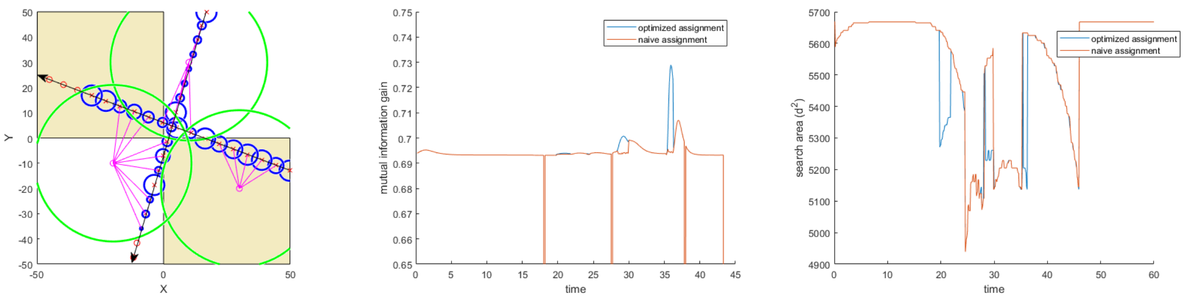
A scenario that demonstrates the utility of considering environmental effects is a simple hand-off. In this scenario, there are two sensors and one target. Initially the target is detected and tracked by one sensor. As it traverses the region of interest, there comes a point where it is advantageous to have the other sensor provide the measurements of the target. Examining the information gain in (31), for cases where the environment is assumed to be homogeneous, $\sigma_\rho^{(ij)}$ and σ_θ are constant and thus the information gain is primarily a function of and inversely proportional to the sensor to target distance ρ_{ij}. Thus for the assumed homogeneous case, the optimal sensor target assignment that maximizes information gain is the one that minimizes ρ_{ij}. If the environment is considered by varying $\sigma_\rho^{(ij)}$ according to estimated environmental losses, the information gain varies with both $\sigma_\rho^{(ij)}$ and ρ_{ij}, leading to possibilities where maximizing information gain dictates a different assignment than the minimal distance assignment. This is illustrated in Fig 3.



(a) Homogeneous environment **(b) Heterogeneous environment**

Fig. 3 Target hand-off compared in a homogeneous environment and a heterogeneous environment. In (b), the target assignment changes earlier compared to (a) in the target track because the environment causes the nearer sensor to get a poorer measurement

In this simulation, the simple hand-off scenario in a homogeneous environment and a known heterogeneous environment is compared. In the case of a homogeneous environment, the behavior is as expected. The optimal assignment is the sensor that is closest to the target. For the heterogeneous environment, it can be seen that assignment changes such that the sensor that does not have to sense through the high impedance environment is assigned to the target even though it is much farther away. This scenario illustrates that a better assignment may be found by modeling



(a) Target track **(b) Information gain** **(c) Search area**

Fig. 4 Three sensors track two targets. (a) Tracks, covariances, and assignments over time. The information gain is compared between the naive assignment and the optimized assignment in (b), showing that the optimized assignment is superior. In (c), the total search fence resulting from each assignment is shown.

environmental effects in the constrained assignment problem, which results in a lower variance state estimate of tracked targets.

To better understand performance differences, further scenarios are considered to evaluate performance of the assignment optimized with the modeled environment versus the assignment naively optimized without considering environmental effects. The first scenario deals with the case where there are fewer targets than sensors. In this scenario, two targets move through a heterogeneous environment containing three sensors. Two assignments are evaluated. The first assignment performs the optimization assuming that the environment is homogeneous and thus naively minimizes the sensor-target distance. The second assignment includes the known heterogeneous environment in its model. The simulation results shown in Fig. 4: the target tracks and assignments Fig. 4a, the information gain in Fig. 4b and the total search area of each assignment are shown in Fig. 4c.

In this scenario, there are a few instances (around 20, 30, and 35 seconds) where the information gain from the optimal assignment is significantly greater than for the naive assignment. The search area comparison shows sudden drops for the optimal assignment at these time steps compared to the naive assignment. This suggests that the optimal assignment utilized the sensor resources in a manner that allowed it to track both targets during those time frames, whereas the naive assignment was only able to track one.

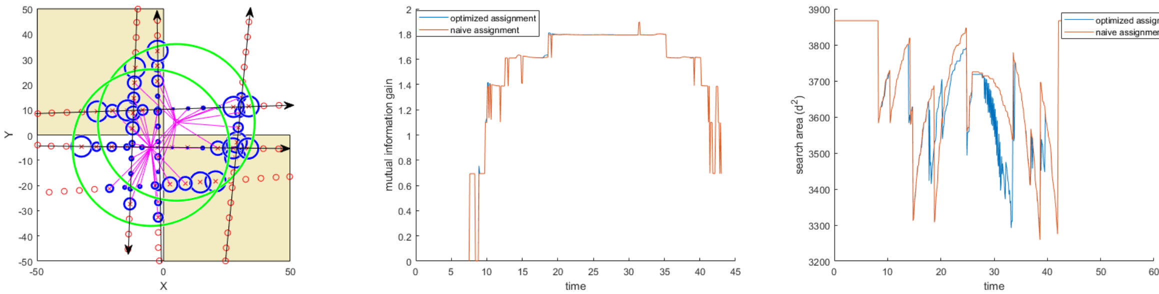
The final scenario considered is the case where there are more targets than sensors. In this case, there are two sensors and six targets. Shown in Fig. 5 is the performance of the two different assignment strategies. As before, there are times in the simulation where the information gain sharply increases and the search area sharply decreases wchich correspond to the tracking of an additional target. As in the case of fewer targets than sensors, the assignment optimization that considers the environment tracks is able to track more targets at certain times than the assignment that does not.

One additional benefit of considering the environment for optimal assignment is that scenarios where the number of targets is large enough that the sensors do not have the resources to track all of them, the environment-considered assignment will track the targets for which it can get the best estimates. This is advantageous because it prevents the use of resources to obtain poor estimates of targets when the resources might be better utilized for search. Using the environment in assignment ensures tracking resources are devoted to targets for which the estimates are most valuable.

V. Conclusion

This paper proposes a formulation of the multiple-target detection and tracking problem. This formulation uses knowledge of signal disruption due to environmental conditions to inform the model of both the detection and tracking functions of a radar. With this updated model, an information filter is run on multiple decentralized sensors to estimate the states of multiple targets simultaneously. The assignment at each time step is optimized to maximize the resulting information gain. The optimization is constrained to assignment solutions that fall within maximum available resources for each sensor. Simulated scenarios demonstrate the utility of considering the environment in the assignment optimization. For cases with multiple sensors and multiple targets, the solution that considered the environment was able to track more targets during the simulation than the solution that did not.

Future work includes implementing a more efficient optimization of the assignment problem. As the number of



(a) Target track **(b) Information gain** **(c) Search area**

Fig. 5 Two sensors track six targets. The tracks, covariances, and assignments over time are shown in (a). The information gain is compared between the naive assignment and the optimized assignment in (b), showing that the optimized assignment is superior. In (c) the total search area resulting from each assignment is shown. The smaller search areas shown in portions of optimized assignment means more targets were being tracked simultaneously.

possible solutions scales exponentially with increasing sensors and targets, brute force optimization quickly becomes intractable. With a better optimization algorithm Monte Carlo simulations could be performed across a wide range of scenarios to better characterize performance differences. Another aspect that could be considered is the case where the environment is not known and one must estimate the environment and target tracks simultaneously. Future work could also investigate mobile sensors optimizing sensor placement and motion to yield better track estimates.

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