AIAA SciTech Forum
8-12 January 2024, Orlando, FL
AIAA SCITECH 2024 Forum

10.2514/6.2024-0325

Lyapunov-based Two-Axis Magnetic Attitude Control of a Rigid Spacecraft

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The attitude control of a rigid spacecraft using two magnetic torque rods is considered. Because the system is underactuated, the attitude error is defined in terms of the reduced-attitude representation. The control objective is the stabilization of the reduced attitude and angular velocity of the spacecraft to a reference trajectory. A control law is proposed and Lyapunov analysis demonstrates that the proposed control law stabilizes the reduced attitude and angular velocity of a rigid spacecraft using only two magnetic torque rods. Numerical simulations illustrate the performance of the control law.

Nomenclature

$\mathcal{I}$	=	Earth-centered inertial reference frame
$\mathcal{B}$	=	body-fixed reference frame
$\boldsymbol{R}$	=	rotation matrix describing rotation from $\mathcal{B}$ to $\mathcal{I}$
$\boldsymbol{\omega}$	=	angular velocity of $\mathcal{B}$ relative to $\mathcal{I}$
$\boldsymbol{\omega}_d$	=	desired angular velocity
$\boldsymbol{b}$	=	unit vector fixed in $\mathcal{B}$
$\boldsymbol{\Gamma}$	=	reduced-attitude representation of $\boldsymbol{R}$
$\boldsymbol{r}_d$	=	desired pointing direction
$\boldsymbol{\tau}$	=	total torque acting on spacecraft
$\boldsymbol{m}$	=	magnetic field generated by torque rods
$\boldsymbol{B}$	=	magnetic field of Earth
Ψ	=	pointing error between $\boldsymbol{\Gamma}$ and $\boldsymbol{r}_d$
$\boldsymbol{e}_r$	=	pointing error vector
$\boldsymbol{e}_\omega$	=	angular velocity error
$\Pi_\perp$	=	orthogonal projection function

I. Introduction

In recent years, there has been a growing trend towards the use of smaller spacecraft due to their increased accessibility to space. This increased accessibility can be attributed to their relatively lower weight, which reduces otherwise prohibitively expensive launch costs. Consequently, the paramount objective when designing such spacecraft is to minimize their weight. In this regard, one key area of research being undertaken to achieve this goal involves reducing the weight allocated to actuation for attitude control. Several approaches have been explored to accomplish this, including reducing the number of actuators used for attitude control and using actuators such as magnetic torque rods that are lightweight. These approaches have required research into new control laws that preserve the attitude control performance of more traditional actuation approaches.

The exploration of reduced actuator usage in research is significant not only due to its potential for weight saving potential in small spacecraft, but also because it can demonstrate the robustness of a spacecraft in the event of actuator failure. Extensive research has addressed attitude control using only two reaction wheels. In [1], a time-varying feedback law is proposed that exponentially stabilizes the attitude of a rigid spacecraft. In [2], it is shown that the angular velocity of a rigid spacecraft can be stabilized with two constant torques. One approach demonstrated in [3] is to stabilize a

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pointing axis of the spacecraft and ignore rotations about that axis. Another area of research evaluates using two control moment gyros instead of reaction wheels. In [4], conditions are defined under which angular velocity damping can occur using two single-gimbal control moment gyros. Full three-axis control was demonstrated in [5] using a backstepping controller under the condition that the initial total angular momentum is zero. In [6], the angular momentum restriction is loosened using the addition of a sliding-mode control to stabilize the underactuated axis. Although extensive research exists on underactuated systems employing more conventional actuators, there is comparatively limited investigation into underactuated control utilizing magnetic actuation.

Magnetic torque rods have been employed on spacecraft from very early on in the space age. The primary advantages of using magnetic actuators are that they are lightweight, small, require no consumables, and are reliable due to their lack of moving parts. One of the earliest uses of magnetic torque rods was dumping momentum from a momentum storage device, like a reaction wheel, as in [7]. Other approaches for this application include [8], which uses the periodicity of orbital motion to analyze the problem, and [9], which uses techniques from optimal control. Magnetic actuation has also traditionally been used for the initial de-tumbling of a spacecraft after deployment. The most common method used is called B-dot control, which uses the rate of change of the magnetic field to determine the actuation torque. Extensive analysis has been performed to validate this control, [10–12]. More recently, there have been efforts to investigate full three-axis attitude control for spacecraft using magnetic actuation. In [13], the problem is represented by a linear time-varying system, which enables controllability analysis to be performed and, for a certain type of orbit, magnetic actuation is shown to make the system controllable. Nonlinear analysis demonstrates attitude control under the disturbance of gravity gradient torque in [14].

While underactuated attitude control and magnetic actuation have been thoroughly investigated separately, there is a lack of research into underactuated magnetic attitude control. Detumbling a satellite with a single magnetic torque rod using a model predictive controller is shown in [15], but the absence of attitude stabilization requires additional actuators. Attitude stabilization with two magnetic torque rods is investigated in [16] using controllability analyses of the linearized dynamics, but the result only holds for a specific attitude reference. Here we show that two magnetic torque rods can control the reduced attitude of a constant angular velocity attitude reference.

The contributions of this paper are as follows: (1) a nonlinear feedback control law for two-axis attitude control of a rigid body actuated by two magnetic torque rods; and (2) Lyapunov analysis guaranteeing that the pointing angle of a spacecraft will converge to any pointing reference that varies constantly, and that the angular velocity of the spacecraft orthogonal to the pointing angle will converge to the angular velocity of the pointing reference. This control law shows that even with a platform that is underactuated and has access to only magnetic actuation, the relevant degrees of freedom for pointing applications can still be controlled.

The paper is organized as follows. The preliminaries and problem statement are given in Section II. The proposed control law and analysis is presented in Section III. Simulations validating the performance of the control law are shown in Section IV, and a conclusion is given in Section V.

II. Preliminaries

A. Spacecraft Attitude Dynamics

Consider the rotational dynamics of a spacecraft modeled as a rigid body. Consider the Earth-centered inertial (ECI) reference frame $\mathcal{I} = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ and a body-fixed reference frame $\mathcal{B} = \{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3\}$ attached to the center of mass of the spacecraft $\mathcal{B}$. The attitude of the spacecraft relative to the inertial frame can be expressed using several parameterizations, such as Euler angles, quaternions, or modified Rodrigues parameters. A rotation matrix $\boldsymbol{R} \in \mathrm{SO}(3)$ is used here to avoid kinematic singularities and unwinding that can result from other representations. The special orthogonal group $SO(3)$ is the group of rigid-body rotations defined as $SO(3) = \{\boldsymbol{R} \in \mathbb{R}^{3\times 3} | \boldsymbol{R}^T\boldsymbol{R} = \mathcal{I}, \det(\boldsymbol{R}) = 1\}$. The rotational equations of motion for the spacecraft are [17]

$$\dot{\boldsymbol{R}} = \boldsymbol{R}\widehat{\omega} \tag{1}$$

$$\boldsymbol{J}\dot{\omega} = \boldsymbol{J}\omega \times \omega + \boldsymbol{\tau}, \tag{2}$$

where $\boldsymbol{J}$ is the spacecraft moment of inertia, $\boldsymbol{\omega}$ is its angular velocity, and $\boldsymbol{\tau}$ is the total external moment on the spacecraft. The hat map $\wedge : \mathbb{R}^3 \to \mathrm{so}(3)$ transforms a vector into a skew-symmetric matrix, i.e., $\widehat{\boldsymbol{a}}\boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$.

B. Reduced Attitude Representation

Assume that the objective of the spacecraft is to point an onboard instrument in a desired direction, given that the instrument is fixed in the body frame. In a pointing application, rotations about the pointing direction are irrelevant; a reduced-attitude representation can be used instead. The pointing direction of the instrument can be described by a vector $\boldsymbol{b}$ on the two-sphere, defined as $\mathbb{S}^2 = \{\boldsymbol{x} \in \mathbb{R}^3 | ||\boldsymbol{x}|| = 1$. Assume that the body-fixed frame is aligned with the principal axes of the spacecraft and that the pointing vector $\boldsymbol{b}$ is aligned with the one of the body-frame axes. Without loss of generality, consider $\boldsymbol{b}$ to be aligned with the third principal axis, i.e., expressed in the body-fixed frame, $\boldsymbol{b} = [0 \quad 0 \quad 1]^T$. In the inertial frame, $\boldsymbol{\Gamma} = \boldsymbol{R}\boldsymbol{b} \in \mathbb{S}^2$ represents the pointing direction of the vector $\boldsymbol{b}$. Because $\boldsymbol{\Gamma}$ is invariant to rotations about $\boldsymbol{b}_3$, it is a reduced-attitude representation of the orientation of the spacecraft [18].

C. Control Input Model

Assume that the spacecraft is actuated solely by two magnetic torque rods orthogonal to $\boldsymbol{b}$ and to each other. That is, the torque rods are aligned with the first two principal axes of the spacecraft $\boldsymbol{b}_1, \boldsymbol{b}_2$, and the control input to each is the strength of the magnetic dipole generated, m_1, m_2. Magnetic torque rods work by generating a magnetic dipole; the interaction of each dipole with Earth's magnetic field induces a moment. The magnetic dipole generated by the control inputs expressed in the body frame is

$$\boldsymbol{m} = \begin{bmatrix} m_1 & m_2 & 0 \end{bmatrix}^T \tag{3}$$

and the resulting moment in the body frame is [17]

$$\boldsymbol{\tau}_{\text{tr}} = \boldsymbol{m} \times \boldsymbol{B} = \begin{bmatrix} m_2 B_3 & -m_1 B_3 & m_1 B_2 - m_2 B_1 \end{bmatrix}^T, \tag{4}$$

where $\boldsymbol{B}$ is the local magnetic field of the Earth expressed in the body frame of the spacecraft.

D. Attitude and Angular Velocity Error States

A function Ψ that defines the pointing error between the spacecraft $\boldsymbol{b}$ and the desired pointing direction $\boldsymbol{R}^T\boldsymbol{r}_d$ is [19]

$$\Psi = 1 - \boldsymbol{b} \cdot \boldsymbol{R}^T \boldsymbol{r}_d, \tag{5}$$

where $\boldsymbol{r}_d$ is the desired pointing direction in the inertial frame. The error function 5 can also be expressed as $1 - \cos\theta$, where θ is the angle between the unit vectors $\boldsymbol{b}$ and $\boldsymbol{R}^T\boldsymbol{r}_d$. The error function is positive definite and has critical points occurring at $\theta = \pm\pi$, [19].

Define the pointing error vector as

$$\boldsymbol{e}_r = \boldsymbol{R}^T \boldsymbol{r}_d \times \boldsymbol{b}. \tag{6}$$

The error vector 6 can be interpreted as a gradient vector field on $\mathbb{S}^2$ induced by the potential function Ψ; $\boldsymbol{e}_r$ vanishes at the critical points of Ψ [19].

Define the angular velocity error as

$$\boldsymbol{e}_\omega = \boldsymbol{\omega} - \boldsymbol{R}^T \boldsymbol{\Omega}_d, \tag{7}$$

where $\boldsymbol{\Omega}_d$ is the desired angular velocity of the spacecraft in the inertial frame and $\boldsymbol{\omega}_d = \boldsymbol{R}^T\boldsymbol{\Omega}_d$ is the angular velocity of the spacecraft expressed in the body frame.

The dynamics of the pointing error are computed by taking the derivative of Ψ, which yields

$$\dot{\Psi} = 0 - \boldsymbol{b}^T \left(\dot{\boldsymbol{R}}^T \boldsymbol{r}_d + \boldsymbol{R}^T \dot{\boldsymbol{r}_d} \right) = -\boldsymbol{b}^T \left(-\boldsymbol{\omega}^\times \boldsymbol{R}^T \boldsymbol{r}_d + \boldsymbol{R}^T (\boldsymbol{\Omega}_d \times \boldsymbol{r}_d) \right). \tag{8}$$

Taking advantage of the triple product identity $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b} \cdot (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b})$ and rearranging Eq. (8) yields

$$\dot{\Psi} = (\boldsymbol{R}^T \boldsymbol{r}_d \times \boldsymbol{b}) \cdot (\boldsymbol{\omega} - \boldsymbol{R}^T \boldsymbol{\Omega}_d) = \boldsymbol{e}_\omega^T \boldsymbol{e}_r, \tag{9}$$

which conveniently contains $\boldsymbol{e}_r$ and $\boldsymbol{e}_w$.

The dynamics of the angular velocity error are computed by taking the derivative of $\boldsymbol{e}_\omega$ as follows:

$$\dot{\boldsymbol{e}}_\omega = \dot{\boldsymbol{\omega}} - \dot{\boldsymbol{R}}^T \boldsymbol{\Omega}_d - \boldsymbol{R}^T \dot{\boldsymbol{\Omega}}_d = \dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^\times \boldsymbol{R}^T \boldsymbol{\Omega}_d - \boldsymbol{R}^T \dot{\boldsymbol{\Omega}}_d = \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{\omega}_d - \dot{\boldsymbol{\omega}}_d. \tag{10}$$

The objective of a control law is to drive both error states, Ψ and $\boldsymbol{e}_\omega$, to zero.

III. Control Design

The angular velocity of the spacecraft about the pointing vector $\boldsymbol{b}$ does not affect the kinematics of the reduced attitude representation. Therefore consideration of only the angular velocity orthogonal to $\boldsymbol{b}$ is required track a pointing angle reference because the reduced-attitude representation is invariant to rotations about itself. The projection of a vector $\boldsymbol{a}$ onto another vector $\boldsymbol{b}$, $\text{proj}_{\boldsymbol{b}}\boldsymbol{a}$, can be expressed in matrix form as $\Pi_{\|}(\boldsymbol{b})\boldsymbol{a}$

$$\Pi_{\|}(\boldsymbol{b}) = \boldsymbol{b}\boldsymbol{b}^T \tag{11}$$

The orthogonal projection of a vector in $\mathbb{R}^3$ can be expressed in matrix form as

$$\Pi_{\perp}(\boldsymbol{b}) = \boldsymbol{I}_3 - \Pi_{\|}(\boldsymbol{b}) = \boldsymbol{I}_3 - \boldsymbol{b}\boldsymbol{b}^T \tag{12}$$

The angular velocity error orthogonal to $\boldsymbol{b}$ can then be expressed as

$$\boldsymbol{e}_{\omega\perp} = \Pi_{\perp}\boldsymbol{e}_{\omega} \tag{13}$$

The projection matrices about the pointing vector $\boldsymbol{b}$ will be denoted as $\Pi_{\|}$ and $\Pi_{\perp}$ for simplicity.

Assume that the attitude dynamics occur on a significantly faster time scale than the orbital dynamics and so variations of the local magnetic field due to translation are neglected (an assumption commonly made in analysis of the B-dot algorithm [15]), therefore $\boldsymbol{B}$ in the inertial reference frame is assumed to be constant. Assume also that the local magnitude field is never orthogonal to $\boldsymbol{b}$, i.e., $B_3 \neq 0$. A common spacecraft pointing objective is to orient a sensor or other instrument directly towards the body that it is orbiting (e.g., an Earth-imaging spacecraft). In these applications, for the instrument to remain correctly aligned, the desired angular velocity of the spacecraft must be equal to the angular velocity of its orbit and, for circular orbits, that angular velocity is constant. Therefore, assume that the angular velocity reference remains constant, i.e., $\boldsymbol{\Omega}_d = 0$.

Consider the following sum of the pointing angle error and the angular velocity error orthogonal to the pointing vector as a candidate Lyapunov function:

$$V = k_p\Psi + T = k_p(1 - \boldsymbol{b}^T\boldsymbol{R}^T\boldsymbol{r}_d) + \frac{1}{2}\boldsymbol{e}_{\omega\perp}^T J\boldsymbol{e}_{\omega\perp}, \tag{14}$$

Taking the derivative of the candidate Lyapunov function V yields

$$\dot{V} = k_p\boldsymbol{e}_{\omega\perp}^T\boldsymbol{e}_r + \boldsymbol{e}_{\omega\perp}^T J\dot{\boldsymbol{e}}_{\omega\perp} = k_p\boldsymbol{e}_{\omega\perp}^T\boldsymbol{e}_r + \boldsymbol{e}_{\omega\perp}^T J\left(\Pi_{\perp}\left[J^{-1}(\omega \times J\omega + J^{-1}\boldsymbol{\tau} - \omega \times \omega_d\right]\right). \tag{15}$$

Under the assumptions stated in Section II.B, the matrices J and $\Pi_{\perp}$ commute, thus

$$\dot{V} = k_p\boldsymbol{e}_{\omega\perp}^T\boldsymbol{e}_r + \boldsymbol{e}_{\omega\perp}^T J\dot{\boldsymbol{e}}_{\omega\perp} = k_p\boldsymbol{e}_{\omega\perp}^T\boldsymbol{e}_r + \boldsymbol{e}_{\omega\perp}^T\Pi_{\perp}\left[\boldsymbol{\omega} \times J\boldsymbol{\omega} + \boldsymbol{\tau} - J\boldsymbol{\omega} \times \boldsymbol{\omega}_d\right] \tag{16}$$

Substituting $\boldsymbol{\omega}_d = \boldsymbol{\omega} - \boldsymbol{e}_{\omega}$ and utilizing the identity $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b} \cdot (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b})$, the expression Eq. (16) becomes

$$\dot{V} = k_p\boldsymbol{e}_{\omega\perp}^T\boldsymbol{e}_r + \boldsymbol{e}_{\omega\perp}^T\boldsymbol{\tau} + \boldsymbol{e}_{\omega}^T(J\boldsymbol{e}_{\omega\perp} \times \omega) + \boldsymbol{e}_{\omega\perp}^T \times (\omega \times J\omega). \tag{17}$$

Consider the following control input to the magnetic torque rods:

$$\boldsymbol{m} = -k_p\Pi_{\|}\frac{1}{\boldsymbol{B}} \times \left(\boldsymbol{e}_r + \boldsymbol{w} \times J\boldsymbol{\omega} + \Pi_{\perp}J\boldsymbol{w}^{\times}\boldsymbol{e}_{\omega}\right) - k_{\omega}\Pi_{\perp}(\boldsymbol{B} \times \Pi_{\perp}\boldsymbol{e}_{\omega}). \tag{18}$$

Note that because the first term is a cross product with a vector parallel to $\boldsymbol{b}$ and the second term is an orthogonal projection relative to $\boldsymbol{b}$, this expression yields a vector orthogonal to $\boldsymbol{b}$. In the body frame, there are nonzero entries corresponding to the directions of the magnetic torque rods and a zero entry corresponding to the direction of $\boldsymbol{b}$. Therefore, $\boldsymbol{m}$ represents a valid expression for the specified control input model. Substituting the torque resulting from the control input in Eq. (18) $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$ into Eq. (17) yields

$$\dot{V} = -k_w B_3^2\|\boldsymbol{e}_{\omega\perp}\|^2 \leq 0 \quad \dot{V} = 0 \quad \text{iff} \quad \boldsymbol{e}_{\omega\perp}\| = 0. \tag{19}$$

Therefore, the proposed control law asymptotically drives the pointing error and angular velocity error orthogonal to the pointing vector to zero for any constantly-varying pointing reference.

IV. Simulation Results

Numerical simulations illustrate the performance of the closed-loop system. A rigid spacecraft is simulated in a circular orbit around Earth. The orbit is specified as an 800km altitude, 60 degree inclined circular orbit with a true anomaly at the initial condition of 90 degrees and the other orbital parameters set to 0. The magnetic field of the Earth was modeled using a dipole model and the gains used in the controller were $k_p = 5 \times 10^{-5}$, and $k_w = 0.1$. The following initial conditions were used for the numerical simulation

$$\boldsymbol{R}_0 = \mathbb{I}^{3\times3}, \quad \omega_0 = \begin{bmatrix}4 & -5 & 2\end{bmatrix}^T \times 10^{-3}$$

with a mass of $5kg$ and a moment of inertia matrix of

$$J = \text{diag}\begin{bmatrix}0.25 & 0.25 & 0.4\end{bmatrix}$$

The reference attitude trajectory supplied to the controller was a nadir-pointing attitude, i.e., pointing towards the Earth's center $\boldsymbol{r}_d = \boldsymbol{r}/\|\boldsymbol{r}\|$, with the reference angular velocity $\boldsymbol{\Omega}_d$ as the angular velocity of the spacecraft's orbit. The given initial conditions require a slewing maneuver to align with the reference attitude at the correct angular velocity. The results of the simulation are plotted below.

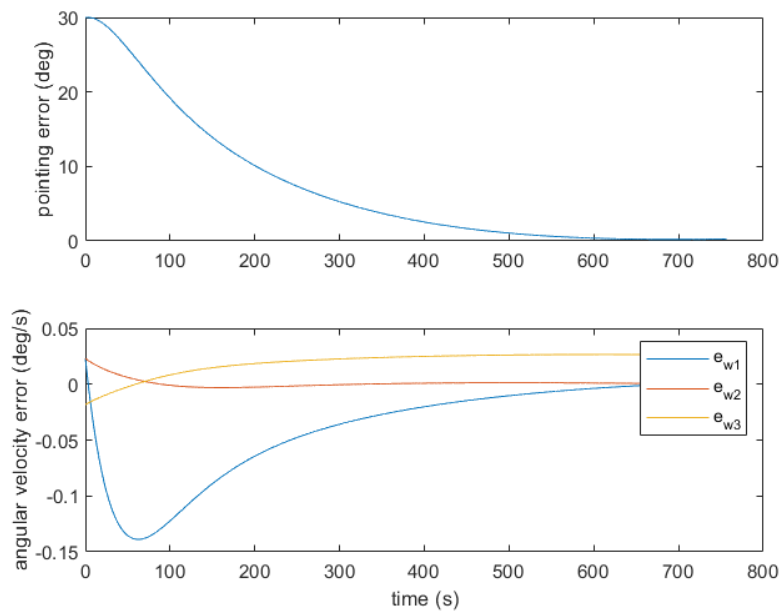

Fig. 1 Magnitude of the pointing error and angular velocity error over a 750-second simulation. The angle of the pointing angle and the angular velocity error orthogonal to the pointing vector both converge to zero.

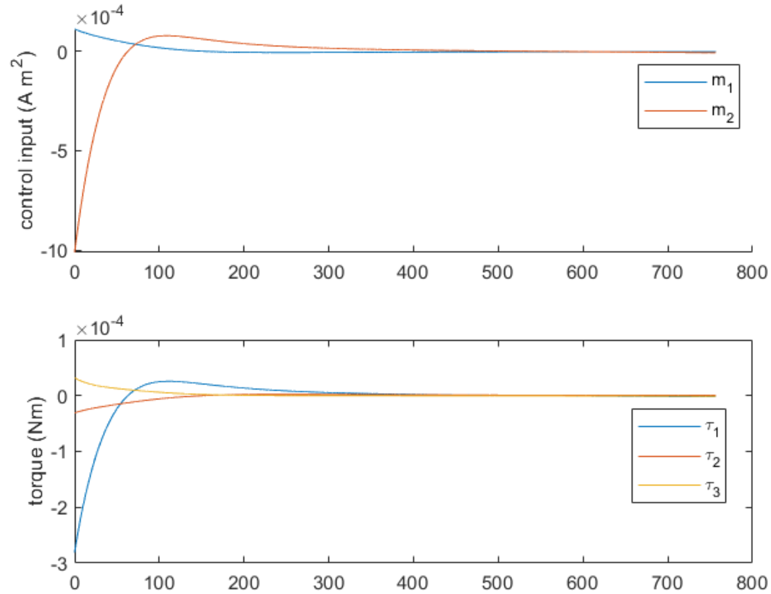

Fig. 2 Control inputs and resulting torques over a 750-second simulation. The actuation torque is on the order of tenths of millinewton meters, which is a magnitude realistic to magnetic torque rods.

Simulation results demonstrate the asymptotic convergence of the error states representing pointing error and angular velocity error orthogonal to the pointing vector to zero. Note that there is some residual angular velocity about the pointing vector, which is akin to a nadir-pointing spacecraft that is rotating about its pointing axis. This outcome is expected because, with only two magnetic torque rods, full attitude stabilization is not possible. However, when the other error states have converged to zero, this residual angular velocity does not affect the pointing angle or its kinematics. Note also that because the magnetic field in the simulation is taken as a function of the spacecraft's position relative to Earth, the simulation suggests that the controller still stabilizes the closed-loop system even when relaxing the assumption of a constant magnetic field.

An additional simulation was performed with a larger initial angular velocity to represent the scenario of detumbling the spacecraft and then aligning it in the desired direction. The same parameters were used except the initial angular velocity, which is now set as $\omega_0 = [-3 \quad -4 \quad 2]^T \times 10^{-2}$. The resulting simulations are shown below.

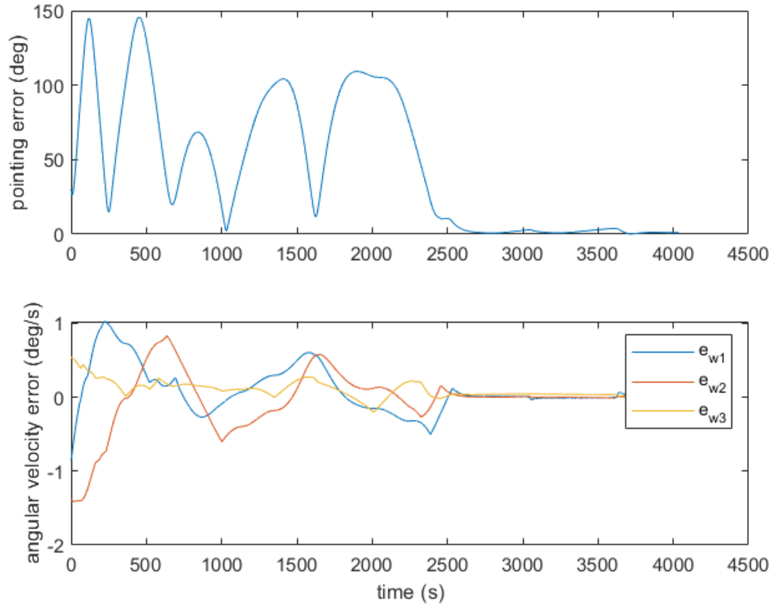

Fig. 3 Magnitude of the pointing error and angular velocity error over a 4000-second detumbling simulation. The angle of the pointing angle and the angular velocity error orthogonal to the pointing vector both converge to zero.

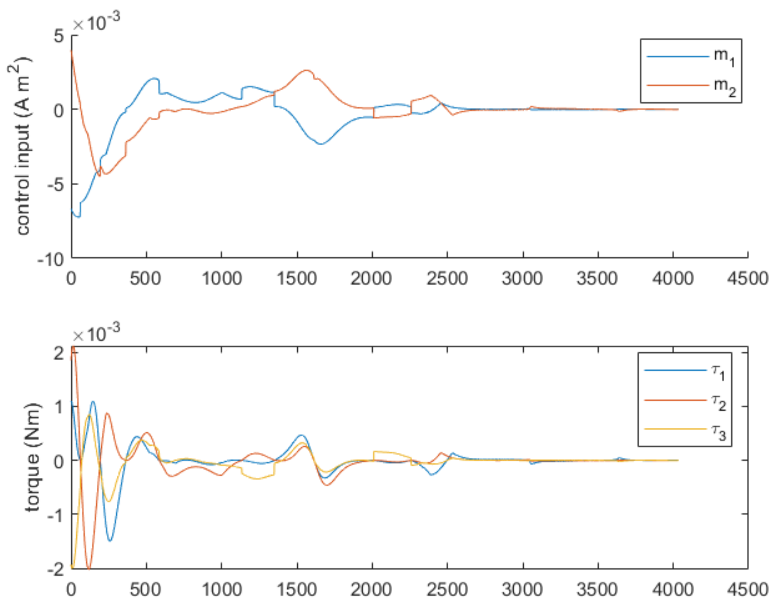

Fig. 4 Control inputs and resulting torques over a 4000-second detumbling simulation. The torque developed by the magnetic torque rods is on the order of millinewton meters.

The simulation results demonstrate controller performance from a more demanding initial condition. Although it takes a significant amount of time, the simulation shows that the controller does eventually remove enough rotational energy from the spacecraft to then successfully track the desired reference.

V. Conclusion

This paper addresses the attitude control problem of a rigid spacecraft utilizing two magnetic torque rods. The underactuated nature of the system necessitates defining the attitude error in terms of the reduced-attitude representation. The primary control objective focuses on stabilizing the reduced attitude and angular velocity of the spacecraft towards a predefined reference trajectory. To achieve this objective, a novel control law is proposed and analyzed using Lyapunov analysis. The theoretical analysis suggests that the proposed control law effectively stabilizes the reduced attitude and angular velocity of the spacecraft using only two magnetic torque rods. Numerical simulations are conducted to validate the performance of the proposed control law. The results of these simulations demonstrate the proposed control law achieving stability of the defined error states. Ongoing research includes performing additional numerical simulations to examine the performance of the controller while relaxing more of the assumptions made for the analysis. ONgoing and future research seeks to include the periodic variations of the local magnetic field corresponding to orbital periods, which make the dynamics non-autonomous and requires additional analysis to characterize. Robustness of the control law to disturbances such as gravity-gradient torque are also of interest.

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