Stabilization of Collective Motion in a Uniform and Constant Flow Field

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Decentralized, cooperative control algorithms enable autonomous sensing platforms to conduct synoptic, adaptive surveys of dynamic spatiotemporal processes. Often these sensing platforms are advected by strong and variable flow fields—i.e., winds in the atmosphere and currents in the ocean. Existing cooperative control algorithms are based on simple models of vehicle motion that do not consider an external flow field. In this paper, we introduce a planar motion model that explicitly incorporates a flow field and, for uniform and constant flows, we provide decentralized control algorithms that stabilize basic motion primitives in the model. The motion primitives include synchronized motion, in which all of the vehicles move in the same direction with arbitrary separation; balanced motion, in which the centroid of vehicle positions is fixed; and circular motion, in with all of the particles travel around a circle with a fixed center. By introducing a virtual particle that serves as reference, we derive a circular-motion algorithm that stabilizes motion around a prescribed center point.

Nomenclature

N	Number of particles
r_k	Position of particle k
$\dot{r}_k$	Velocity of particle k
v_k	Speed of particle k
f_k	Flow velocity experienced by particle k
β	Magnitude of flow field
θ_k	Orientation of particle k's velocity relative to the flow
γ_k	Orientation of particle k's inertial velocity
c_k	Center of circle traversed by particle k
ω_0	Constant angular rate
P	$N \times N$ projector matrix
P_k	kth row of matrix P
K	Control gain
i	Imaginary unit

Subscript

k	Particle index, $1, \ldots, N$

I. Introduction

A network of autonomous vehicles in the air or sea provides a distributed and adaptable sensory array applicable to, for example, in-situ environmental assessment, target surveillance, and remote sensing. Feedback coordination of the trajectories of multiple sensor platforms enhances the performance of the entire array by eliminating redundant sampling and regulating spatiotemporal separation between the array elements.[6] Recent progress in the design of theoretically justified coordination algorithms has focused on simplified models

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of the vehicle dynamics.[3,11,12] Each vehicle is modeled as a Newtonian particle that moves at constant speed subject to a gyroscopic, shape control. Decentralized control algorithms exist to stabilize parallel and circular motion and symmetric patterns on convex curves with all-to-all or limited communication.[9,11,12] In a symmetric formation, synchronization and balancing controls combine to regulate the particle spacing in a moving formation.

The motion of autonomous sensing platforms in the air and sea is impeded by strong and variable currents. However, the feedback coordination of multiple autonomous vehicles in strong and variable currents has not yet been fully addressed by theoretical methods. Some heuristic results exist for a single vehicle in moderate flow. Davis *et al.* describe a minimum-time routing strategy, based on Snell's law, for an underwater glider in a time-invariant and known flow.[1] Inanc *et al.* describe a nonlinear-optimization technique to generate glider trajectories that minimize travel time and/or energy consumption in a time-varying and known flow.[2] McGee *et al.* study UAV path planning (with an upper bound on the vehicle turning rate) in a spatially uniform, time-invariant and known flow.[7] Note, holding station in a uniform flow is equivalent to tracking a target that moves at a constant speed—a problem studied in the UAV literature.[4,5] Rysdyk considers estimation and control strategies that support tracking a target with a constant line-of-sight in an unknown flow.[10]

In this paper, we provide decentralized control algorithms to stabilize collective motion in a system of self-propelled particles subject to an external flow field. The paper presents a direct extension of the flow-free framework established previously by Sepulchre *et al.*[11,12] We assume that the flow field is uniform and constant, and that the particles move at a constant speed relative to the flow. We also assume that the magnitude of the flow field never exceeds the magnitude of the particle speed relative to the flow. These assumptions are imposed to obtain preliminary results; the assumption of spatial uniformity has been relaxed in ongoing work. We focus here on the case of all-to-all communication; the extension to limited communication that is possibly time-varying and/or directed follows the framework in Sepulchre *et al.*[11]

Under the simplifying assumptions, we provide algorithms to stabilize several motion primitives, including synchronized motion with arbitrary particle separation, balanced motion with a fixed position centroid, and circular motion around a fixed center. We also provide a circular-motion algorithm that parametrizes the steady-state center of the motion. As is the case for the particle model without flow, these motion primitives can be combined to form a control framework for designing complex sampling trajectories in the presence of flow. For example, a decentralized control that regulates particle spacing around a circular formation is proposed elsewhere.[8] Extending the framework to non-uniform and time-varying flows is the subject of ongoing work and not presented here.

The paper outline is as follows. In Section II we introduce a self-propelled particle model that explicitly incorporates an external flow field. In Section III we provide decentralized control algorithms to stabilize collective motion in the presence of flow, including synchronized, balanced, and circular motion. In Section IV, we summarize the results and provide indications of ongoing and future work.

II. Model

Previous work in this area has focused on a self-propelled particle model in which N point masses move at unit speed in an inertial plane. The position of the kth particle is denoted by r_k, $k \in \{1, \ldots, N\}$, and the velocity is $\dot{r}_k$.[a] In complex notation, the velocity is $\dot{r}_k = e^{i\theta_k}$, where $\theta_k \in S^1$ is the orientation or phase angle. Each particle is subject to a state-feedback control u_k. The flow-free particle model is

$$\begin{aligned} \dot{r}_k &= e^{i\theta_k} \\ \dot{\theta}_k &= u_k. \end{aligned} \tag{1}$$

The model (1) is illustrated in Figure 1(a).

We explicitly incorporate a flow field in the particle model (1) by introducing a drift vector field, $f_k \in \mathbb{C}$. The particle model with flow is

$$\begin{aligned} \dot{r}_k &= f_k + e^{i\theta_k} \\ \dot{\theta}_k &= u_k. \end{aligned} \tag{2}$$

[a] We will drop the subscript and use bold to represent an $N \times 1$ matrix, e.g, $\boldsymbol{r} \triangleq [r_1 \cdots r_N]^T$. Note, we identify the $\mathbb{R}^2$ plane with the complex $\mathbb{C}$ plane to facilitate our analysis. The standard inner product in $\mathbb{R}^2$ is represented in $\mathbb{C}$ by $\langle x, y\rangle = \mathrm{Re}\{\bar{x}y\}$, where $x, y \in \mathbb{C}$ and $\bar{x}$ denotes the complex conjugate of x.

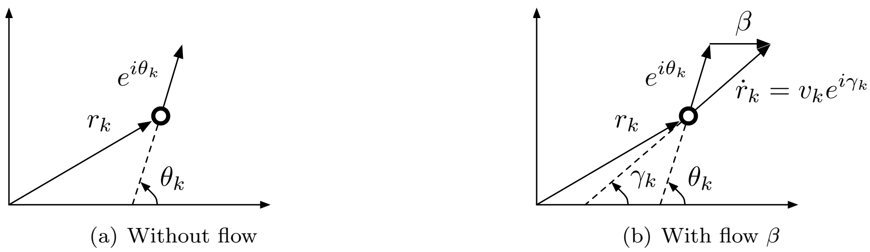


Figure 1. Coordinates and notation for self-propelled particle models.

In general, f_k may vary in time and space, i.e. $f_k = f_k(t)$ and $f_k \neq f_j$.

In this paper, we study a special case of (2) in which the flow is uniform in space and constant in time. We assume that the magnitude of the flow is less than one. Without loss of generality, we align the positive real axis of the inertial reference frame with the direction of the flow. Let $\beta \in \mathbb{R}$ denote the magnitude of the flow in this frame, where $|\beta| < 1$. The particle model with bounded, uniform and constant flow is

$$\begin{aligned} \dot{r}_k &= \beta + e^{i\theta_k} \\ \dot{\theta}_k &= u_k. \end{aligned} \tag{3}$$

This model (3) is illustrated in Figure 1(b).

In (3), the magnitude of the particle inertial velocity depends on the phase angle θ_k. Let $v_k \in \mathbb{R}$ and $\gamma_k \in S^1$ denote, respectively, the magnitude and orientation of the inertial velocity, i.e.,

$$v_k e^{i\gamma_k} = \beta + e^{i\theta_k}. \tag{4}$$

Since, by assumption, $|\beta| < 1$, we observe that $v_k > 0$. The magnitude v_k is

$$\begin{aligned} v_k &= \sqrt{(\beta + e^{i\theta_k})(\beta + e^{-i\theta_k})} \\ &= \sqrt{1 + \beta^2 + 2\beta\cos\theta_k}. \end{aligned} \tag{5}$$

However, we would like to express v_k in terms of γ_k instead of θ_k. Using Figure 1(b), we observe that

$$\sin\theta_k = v_k \sin\gamma_k \tag{6}$$

$$\cos\theta_k = v_k \cos\gamma_k - \beta. \tag{7}$$

Substituting (7) into (5) and rearranging the result yields a quadratic equation in v_k,

$$v_k^2 - 2\beta\cos\gamma_k v_k + \beta^2 - 1, \tag{8}$$

which can be solved (using the positive root, since $v_k > 0$), to obtain

$$v_k = \beta\cos\gamma_k + \sqrt{1 - \beta^2\sin^2\gamma_k}. \tag{9}$$

The orientation γ_k is defined as

$$\gamma_k = \arg\{\beta + \cos\theta_k + i\sin\theta_k\} = \mathrm{atan}\left(\frac{\sin\theta_k}{\beta + \cos\theta_k}\right). \tag{10}$$

Differentiating the expression,

$$\tan\gamma_k = \frac{\sin\theta_k}{\beta + \cos\theta_k}, \tag{11}$$

with respect to time and solving for $\dot{\gamma}_k$, we obtain

$$\dot{\gamma}_k = (\sin^2\gamma_k + \cos\gamma_k \sin\gamma_k \cot\theta_k) u_k. \tag{12}$$

Substituting (6) and (7) into (12) yields

$$\dot{\gamma}_k = (1 - \beta v_k^{-1} \cos\gamma_k) u_k \triangleq \nu_k. \tag{13}$$

We view $\nu_k \in \mathbb{R}$ as a control input, since given ν_k, we can solve for u_k and integrate the model (3). We rewrite (3) as

$$\begin{aligned} \dot{r}_k &= v_k e^{i\gamma_k} \\ \dot{\gamma}_k &= \nu_k, \end{aligned} \tag{14}$$

where v_k is defined in (9). We use the particle model (14) in the design of our feedback control algorithms. It represents a self-propelled particle model in which the particle speed v_k depends on the orientation, γ_k, of the particle velocity vector.

III. Results

III.A. Synchronization and Balancing

Two motion primitives of the particle model (1) are *synchronized* and *balanced* motions. In synchronized motion, all of the particles move in the same direction with arbitrary separation, which implies all of the particle phases are equal. In balanced motion, the centroid of particle positions, $p_r \triangleq (1/N)\sum_{j=1}^N r_j$, is fixed, which implies that the quantity $p_\theta \triangleq (1/N)\sum_{j=1}^N e^{i\theta_j} = \dot{p}_r$ is equal to zero.[12] Note, $|p_\theta| = 1$ for synchronized motion. Synchronized and balanced motions can be stabilized using gradient control with respect to the potential $(1/2)|p_\theta|^2$.[12]

In the particle model (14), synchronized motion of the particles in an inertial frame corresponds to the maximum of the potential

$$U(\boldsymbol{\gamma}) \triangleq \frac{1}{2}|p_\gamma|^2, \tag{15}$$

where

$$p_\gamma \triangleq \frac{1}{N}\sum_{j=1}^N e^{i\gamma} \tag{16}$$

is the centroid of the phasors $e^{i\gamma_k}$, $k = 1, \ldots, N$. The following proposition is a consequence of [12, Theorem 1].

Proposition 1. *The particle model (14) with flow speed $|\beta| < 1$ and the gradient control*

$$\nu_k = -K\frac{\partial U}{\partial \gamma} = -K\langle p_\gamma, ie^{i\gamma_k}\rangle,\ K < 0, \tag{17}$$

forces convergence of all solutions to the critical set of U. The set of synchronized motions are asymptotically stable and every other equilibrium is unstable.

Proposition 1 provides an algorithm to stabilize synchronized motion in a uniform and constant flow field. We stabilize solutions of (14) that exhibit balanced motion by considering the potential

$$V(\boldsymbol{r}, \boldsymbol{\gamma}) = \frac{1}{2}|p_{\dot{r}}|^2, \tag{18}$$

where

$$p_{\dot{r}} \triangleq \frac{1}{N}\sum_{j=1}^N v_j e^{i\gamma_j} \tag{19}$$

is the centroid of the particle velocities. The time-derivative of V along solutions of (14) is

$$\dot{V} = \langle p_{\dot{r}}, \dot{p}_{\dot{r}}\rangle = \sum_{j=1}^N \langle p_{\dot{r}}, \dot{v}_j e^{i\gamma_j} + v_j i e^{i\gamma_j}\dot{\gamma}_j\rangle, \tag{20}$$

where we obtain

$$\dot{v}_k = \frac{-\beta\sin\gamma_k}{\sqrt{1-\beta^2\sin^2\gamma_k}} v_k \dot{\gamma}_k \tag{21}$$

by differentiating (9). Substituting (21) into (20) yields

$$\dot{V} = \sum_{j=1}^{N} \langle p_{\dot{r}}, (\delta_j + i)e^{i\gamma_j} \rangle v_j \nu_j, \tag{22}$$

where $\delta_k \in \mathbb{R}$ is

$$\delta_k \triangleq \frac{-\beta \sin\gamma_k}{\sqrt{1-\beta^2 \sin^2\gamma_k}}. \tag{23}$$

Lyapunov analysis leads to the following result.

Theorem 1. *The particle model (14) with flow speed $|\beta| < 1$ and the control*

$$\nu_k = -K\langle p_{\dot{r}}, (\delta_k + i)e^{i\gamma_k} \rangle v_k, \; K > 0, \tag{24}$$

where δ_k is given by (23), asymptotically stabilizes the set of balanced motions.

Proof. Substituting (24) into (22) yields

$$\dot{V} = -K \sum_{j=1}^{N} \langle p_{\dot{r}}, (\delta_j + i)e^{i\gamma_j} \rangle^2 v_j^2 \leq 0. \tag{25}$$

By the invariance principle, all of the solutions of (14) with the control (24) converge to the largest invariant set, Λ, in which

$$\langle p_{\dot{r}}, (\delta_k + i)e^{i\gamma_k} \rangle \equiv 0 \tag{26}$$

In this set, $\dot{\gamma}_k = 0$ and $\dot{v}_k = 0$, which implies $p_{\dot{r}}$ is constant. Since $\delta_k + i \neq 0$, then the invariance condition (26) is satisfied for all $k = 1, \ldots, N$, only when $p_{\dot{r}} = 0$. □

III.B. Circular Formations

In the absence of flow, i.e., using the model (1), setting $u_k = \omega_0$, where $\omega_0 \neq 0$ is constant, drives particle k around a circle of radius ω_0^{-1} and fixed center,[12]

$$c_k \triangleq r_k + \omega_0^{-1} i \frac{\dot{r}_k}{|\dot{r}_k|}. \tag{27}$$

In the presence of uniform and constant flow, we have the following result.

Lemma 1. *The model (14) with flow speed $|\beta| < 1$ and the control*

$$\nu_k = \omega_0 v_k \tag{28}$$

drives particle k around a circle of radius ω_0^{-1} centered at $c_k(t) = r_k(0) + \omega_0^{-1} i e^{i\gamma_k(0)}$.

Proof. We derive the control ν_k that steers the particle around a circle of radius ω_0^{-1} by differentiating (27) along solutions of (14). This results in

$$\dot{c}_k = v_k e^{i\gamma_k} - \omega_0^{-1} e^{i\gamma_k} \nu_k = (v_k - \omega_0^{-1}\nu_k)e^{i\gamma_k}. \tag{29}$$

Substituting ν_k defined in (28) into (29) yields $\dot{c}_k = 0$, which completes the proof. □

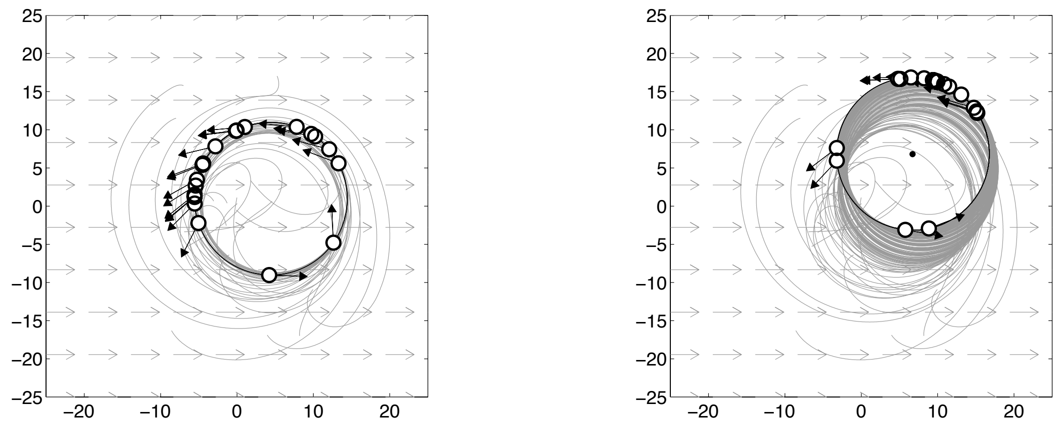
A *circular formation* is a solution of the particle model (14) in which all of the particles orbit the same circle in the same direction. In a circular formation, $c_k = c_j$ for all pairs j and k, which implies that a circular formation satisfies the condition[12]

$$P\boldsymbol{c} = 0, \tag{30}$$

where

$$P = \text{diag}\{\mathbf{1}\} - \frac{1}{N}\mathbf{1}\mathbf{1}^T \tag{31}$$

projects $\mathbb{C}^N$ to the subspace complementary to the span of $\mathbf{1} \triangleq [1 \cdots 1]^T \in \mathbb{R}^N$.



(a) Without symmetry breaking (b) With symmetry breaking

Figure 2. Stabilization of circular motion with $N = 20$ and flow speed, β, equal to 75% of the particle speed relative to the flow. Without symmetry breaking, the particles converge to a circular formation with an arbitrary center. With symmetry breaking, the particles converge to a circular formation with center equal to the reference point $c_0 = 6.7 + i6.8$.

We derive a decentralized control that stabilizes a circular formation by considering the potential

$$S(\boldsymbol{r}, \boldsymbol{\gamma}) \triangleq \frac{1}{2}\langle \boldsymbol{c}, P\boldsymbol{c}\rangle. \tag{32}$$

Note $S \geq 0$, with equality only when $\boldsymbol{c} = c_0\mathbf{1}$, $c_0 \in \mathbb{C}$. The time derivative of S along solutions of (14) is

$$\dot{S} = \sum_{j=1}^{N}\langle \dot{c}_j, P_j\boldsymbol{c}\rangle = \sum_{j=1}^{N}\langle e^{i\gamma_j}, P_j\boldsymbol{c}\rangle(v_j - \omega_0^{-1}\nu_j), \tag{33}$$

where P_k denotes the kth row of P. The following result provides a control algorithm to stabilize circular formation in a uniform and constant flow. It extends [12, Theorem 2], which provides a circular-formation algorithm that applies only in the absence of flow.

Theorem 2. *All solutions of the particle model (14) with flow speed $|\beta| < 1$ and the control*

$$\nu_k = \omega_0(v_k + K\langle P_k\boldsymbol{c}, e^{i\gamma_k}\rangle),\ K > 0, \tag{34}$$

converge to a circular formation with radius ω_0^{-1} and direction determined by the sign of ω_0.

Proof. The potential S is positive definite and proper in the space of relative circle centers. Substituting (34) into (33) yields

$$\dot{S} = -K\sum_{j=1}^{N}\langle P_k\boldsymbol{c}, e^{i\gamma_k}\rangle^2 \leq 0. \tag{35}$$

By the invariance principle, all of the solutions of (14) with control (34) converge to the largest invariant set, Λ, in which

$$\langle P_k\boldsymbol{c}, e^{i\gamma_k}\rangle \equiv 0. \tag{36}$$

In this set, $\dot{\gamma}_k = \omega_0 v_k$ and $\dot{c}_k = 0$. Therefore, in order to satisfy the invariance condition, (36), all of the solutions in Λ must satisfy $P\boldsymbol{c} = 0$, which is the circular-formation condition. Application of Lemma 1 completes the proof. □

We illustrate this result in Figure 2(a), for $N = 20$, $\beta = 0.75$, $K = 0.01$, and $\omega_0 = 0.1$.

III.C. Symmetry Breaking

The control algorithm described in Theorem 2 depends only on relative positions, $r_k - r_j$. Consequently, it preserves a symmetry of the closed-loop particle model that engenders invariance to rigid translation of the collective. This implies the steady-state center of the circle depends on initial conditions. For applications in path-planning for autonomous vehicles, there exists the need to specify the steady-state center of the vehicle formation in the presence of flow. We describe below a symmetry-breaking algorithm that provides this capability.

Following Sepulchre *et al.*,[11] we introduce a virtual particle $k = 0$ that serves as a reference. The virtual particle dynamics,

$$\begin{aligned} \dot{r}_0 &= v_0 e^{i\gamma_0} && (37) \\ \dot{\gamma}_0 &= \omega_0 v_0, && (38) \end{aligned}$$

where $\omega_0 \neq 0$, are independent of the dynamics of the particles; they drive particle 0 in a circle with fixed center $c_0 = r_0(0) + \omega_0^{-1} i e^{i\gamma_0(0)}$. The virtual-particle states are available to a subset of the particles, called informed particles. Let $a_{k0} = 1$ if particle k is an informed particle and $a_{k0} = 0$ otherwise.

Consider augmenting the potential S defined in (32) with the quadratic potential[11]

$$S_0 = \frac{1}{2} \sum_{j=1}^{N} a_{j0} |c_j - c_0|^2, \tag{39}$$

which is minimized when $c_j = c_0$ for all $\{j \mid j \in 1, \ldots, N,\ a_{j0} = 1\}$. The time-derivative of $\tilde{S} \triangleq S + S_0$ along solutions of (14) is

$$\dot{\tilde{S}} = \sum_{j=1}^{N} \left(\langle e^{i\gamma_j}, P_j \boldsymbol{c} \rangle + a_{j0} \langle e^{i\gamma_j}, c_j - c_0 \rangle \right) (v_j - \omega_0^{-1} \nu_j) \tag{40}$$

This leads to the following result, illustrated in Figure 2(b).

Corollary 1. *Let $c_0 = r_0(0) + \omega_0^{-1} i e^{i\gamma_0(0)}$ be the fixed reference provided by the virtual particle $k = 0$ and let a_{k0}, $k = 1, \ldots, N$, equal one if particle k is informed of this reference and zero otherwise. If there is at least one informed particle and no more than $N - 1$ informed particles, then all solutions of the particle model (14) with the control*

$$\nu_k = \omega_0 (v_k + K(\langle e^{i\gamma_k}, P_k \boldsymbol{c} \rangle + a_{k0} \langle e^{i\gamma_k}, c_k - c_0 \rangle)),\ K > 0, \tag{41}$$

converge to a circular formation with radius ω_0^{-1}, direction determined by the sign of ω_0, and center c_0.

Proof. With the control (41), the time-derivative of the augmented potential $\tilde{S}$ satisfies

$$\dot{\tilde{S}} = -K \sum_{j=1}^{N} (\langle e^{i\gamma_k}, P_k \boldsymbol{c} \rangle + a_{k0} \langle e^{i\gamma_k}, c_k - c_0 \rangle)^2 \leq 0. \tag{42}$$

By the invariance principle, all solutions converge to the largest invariant set, Λ, for which

$$\langle e^{i\gamma_k}, P_k \boldsymbol{c} \rangle + a_{k0} \langle e^{i\gamma_k}, c_k - c_0 \rangle \equiv 0 \tag{43}$$

for $k = 1, \ldots, N$. In this set, $\dot{\gamma}_k = \omega_0 v_k$ and $\dot{c}_k = 0$. For $a_{k0} = 0$, then the invariance condition (44) is satisfied only if $P_k \boldsymbol{c} = 0$. This implies $\boldsymbol{c}$ is in the span of $\mathbf{1}$, i.e. $c_k = c_j$ for all pairs k and j. For $a_{k0} = 1$, the invariance condition becomes

$$\langle e^{i\gamma_k}, c_k - c_0 \rangle \equiv 0, \tag{44}$$

which holds only if $c_k = c_0$. This implies $c_k = c_0 \mathbf{1}$, which completes the proof. □

IV. Conclusion

Distributed sensing with multiple, mobile platforms requires cooperative-control algorithms that generate coordinated sampling trajectories in the presence of strong and variable flow fields. The design of these algorithms is based on simple models of platform motion that often ignore the presence of flow. In this paper, we introduce a self-propelled particle model that explicitly incorporates the presence of a uniform and constant flow field. We provide decentralized control algorithms that stabilize synchronized, balanced, and circular motions. We also provide a symmetry-breaking control that parametrizes the steady-state center of the circular motion. These motion primitives will be essential in constructing a cooperative-control framework for autonomous and distributed sensing in the presence of flow. In ongoing research, we aim to extend this framework to non-uniform and possibly time-varying flows.

References

[1] R. E. Davis, N. E. Leonard, and D. M. Fratantoni. Routing strategies for underwater gliders. Submitted to *Deep-Sea Research II.*

[2] T. Inanc, S. C. Shadden, and J. E. Marsden. Optimal trajectory generation in ocean flows. In *Proc. 2005 Amer. Control Conf.*, pages 674–679, Portland, Oregon, June 2005.

[3] E. W. Justh and P. S. Krishnaprasad. Equilibria and steering laws for planar formations. *Systems and Control Letters*, 52(1):25–38, 2004.

[4] D. J. Klein and K. A. Morgansen. Controlled collective motion for trajectory tracking. In *Proc. 2006 American Control Conf. (electronic)*, Minneapolis, Minnesota, June 2006. (7 pages).

[5] E. Lalish, K. A. Morgansen, and T. Tsukamaki. Oscillatory control for constant-speed unicycle-type vehicles. In *Proc. 46th IEEE Conf. Decision and Control*, pages 5246–5251, New Orleans, Louisiana, December 2007.

[6] N. E. Leonard, D. A. Paley, F. Lekien, R. Sepulchre, D. M. Fratantoni, and R. E. Davis. Collective motion, sensor networks and ocean sampling. *Proc. IEEE*, 95(1):48–74, 2007.

[7] T. G. McGee, S. Spry, and J. K. Hedrick. Optimal path planning in a constant wind with a bounded turning rate. In *Proc. AIAA Conf. Guidance, Navigation, and Control (electronic)*, San Francisco, California, August 2005. (11 pages).

[8] D. A. Paley. Cooperative control of an autonomous sampling network in an external flow field. To appear in *Proc. 47th IEEE Conf. Decision and Control*, invited session on "Autonomous exploration and remote sensing".

[9] D. A. Paley, N. E. Leonard, and R. Sepulchre. Stabilization of symmetric formations to motion around convex loops. *Systems and Control Letters*, 57(3):209–215, 2008.

[10] R. Rysdyk. Unmanned aerial vehicle path following for target observation in wind. *J. Guidance, Control, and Dynamics*, 29(5):1092–1100, 2006.

[11] R. Sepulchre, D. A. Paley, and N. E. Leonard. Stabilization of planar collective motion with limited communication. *IEEE Trans. Automatic Control*, accepted.

[12] R. Sepulchre, D. A. Paley, and N. E. Leonard. Stabilization of planar collective motion: All-to-all communication. *IEEE Trans. Automatic Control*, 52(5):811–824, 2007.